Algorithmic Decision Theory and Preference-based Optimization

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ADT 2009

1st International Conference on Algorithmic Decision Theory

Venice, Italy, October 21-23, 2009

- home
- location
- committees
- sponsors
- Submissions and proceedings

ADT2009

1st INTERNATIONAL CONFERENCE ON ALGORITHMIC DECISION THEORY

An Interdisciplinary forum on:

- Uncertainty and Robustness in Decision Making
- Preferences in Reasoning and Decision
- Decision Theoretic Artificial Intelligence
- Learning and Knowledge Extraction

The COST Action IC0602 and the EURO Working Group on Preferences are proud to announce the 1st International Conference on Algorithmic Decision Theory. A new unique event aiming to put together researchers and practitioners coming from different fields such as Decision Theory, Discrete Mathematics, Theoretical Computer Science and Artificial Intelligence in order to improve decision support in the presence of massive data bases, combinatorial structures, partial and/or uncertain information and distributed, possibly inter-operating decision makers. Such problems arise in several real-world decision making problems such as humanitarian logistics, epidemiology, risk assessment and management, e-government, electronic commerce, and the implementation of recommender systems. Contributions to the conference are sought in areas including:

ADT 2011

2nd International Conference on Algorithmic Decision Theory

DIMACS, Rutgers University, October 26 - 28, 2011

- Home Page
- Proceedings
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- Invited Speakers
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- Call for Papers
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- Co-located Events
- Link to ADT2009

2nd International Conference on Algorithmic Decision Theory

The Second International Conference on Algorithmic Decision Theory (ADT) will be held at DIMACS in the Fall of 2011. The conference will be broad in scope, covering all aspects of the growing field of ADT. It will involve researchers from such disparate fields as decision theory, discrete mathematics, theoretical computer science, economics, and artificial intelligence, aiming to improve decision support in the presence of massive data bases, partial and/or uncertain information, and distributed decision makers. It will emphasize real-world decision problems involving fundamental problems of the world's societies: climate change, environmental protection, energy use, health care, financial management, natural disasters, terrorism, etc.

The 1st International Conference on Algorithmic Decision Theory (ADT 2009) was a clear success and demonstrated the demand for establishing a permanent scientific forum for this community. The Second International Conference on Algorithmic Decision Theory will include tutorials, key invited speakers each day, and a call for papers and posters. The Second International Conference on Algorithmic Decision Theory will be a major step in focusing researchers and students from multiple disciplines on the task of developing tools for dealing with the complex decision making problems facing global society.

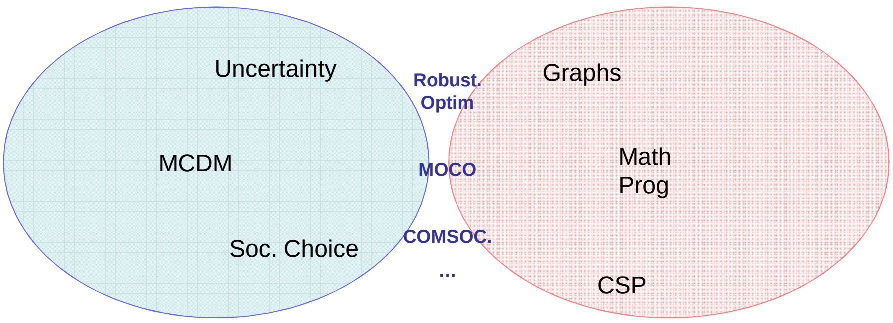
ADT 2011 will be held at the DIMACS Center in Piscataway, New Jersey, a major multidisciplinary research center based at Rutgers University and located close to New York City and Philadelphia.

Enjoy the nearby attractions of New York City and the beautiful Fall colors of New Jersey.

Preference-based optimization

Decision Theory

Discrete and continuous optimization



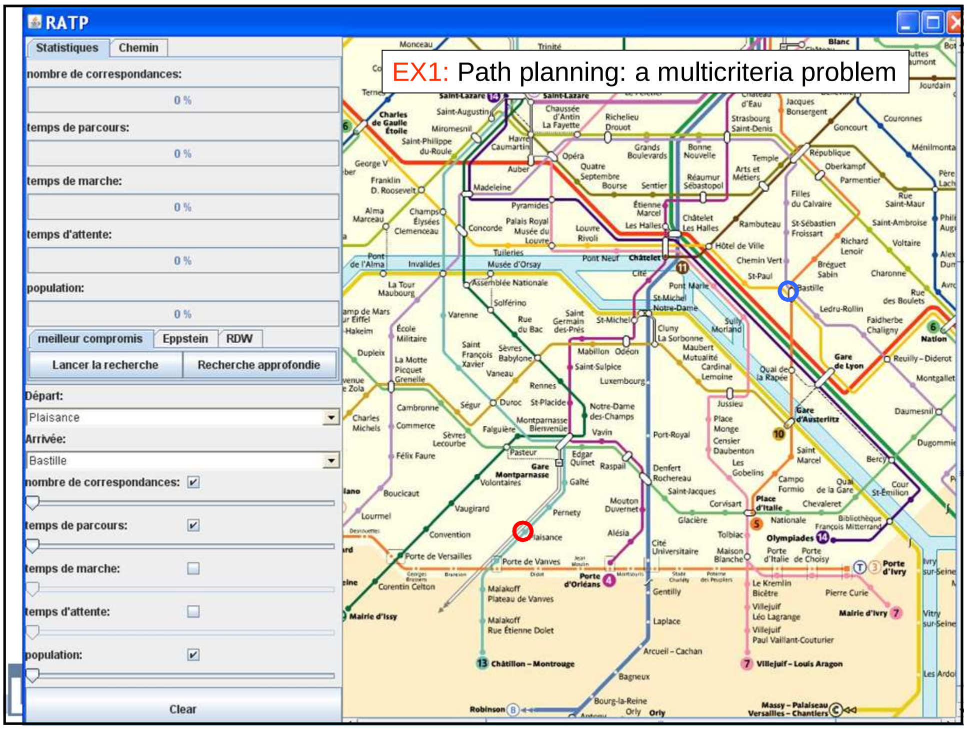
Preference Modelling
Aggregation, Decision

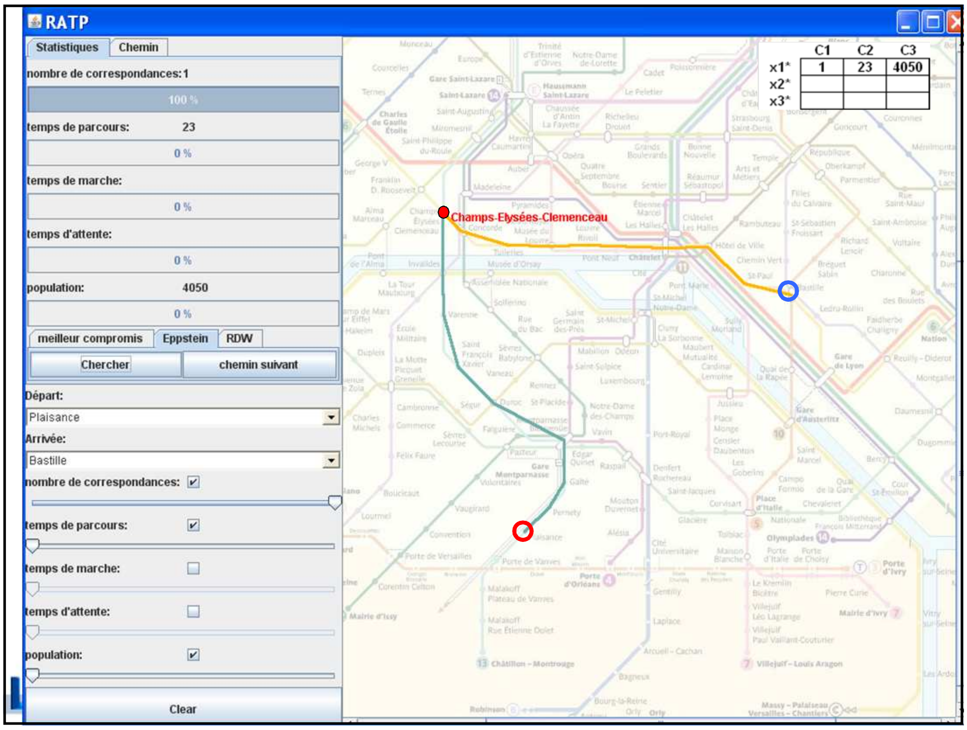
↔

Complexity, algorithms,
approximation,

I) Examples and motivations

- Compromise search in multiobjective optimization
- Equity in multiagent assignment problems
- Robustness in optimization under uncertainty
- Risk-averse optimization

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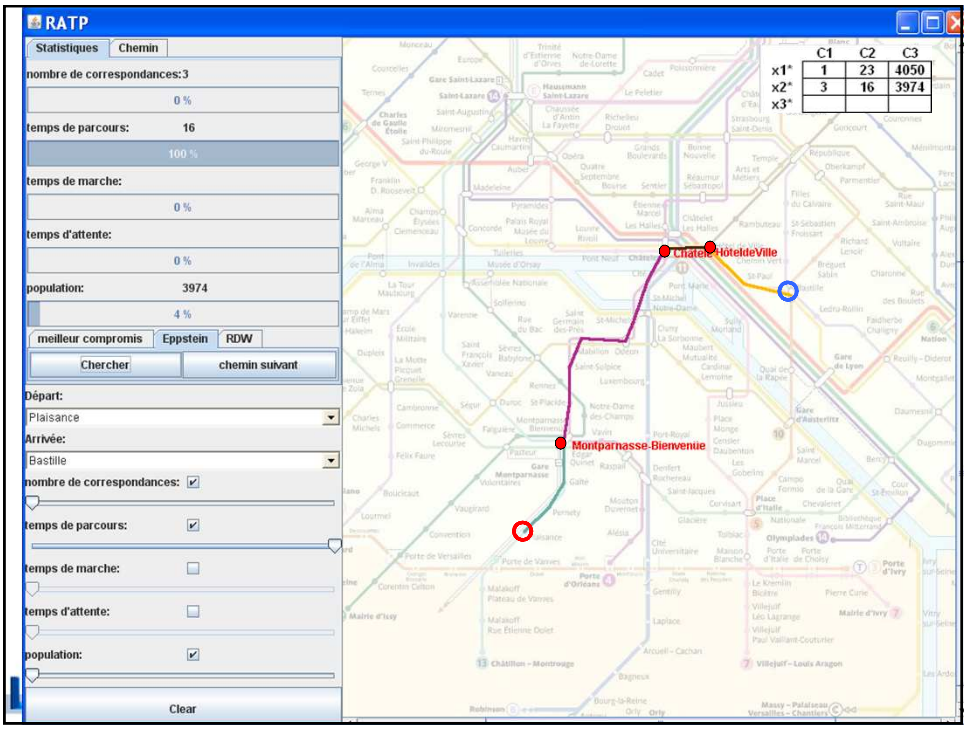
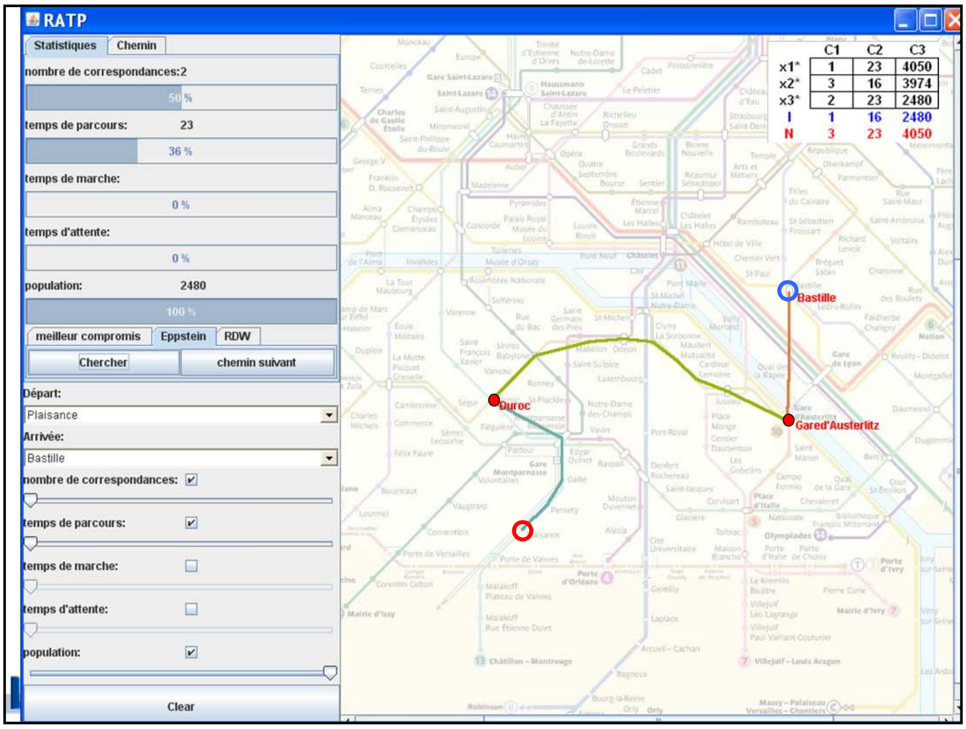
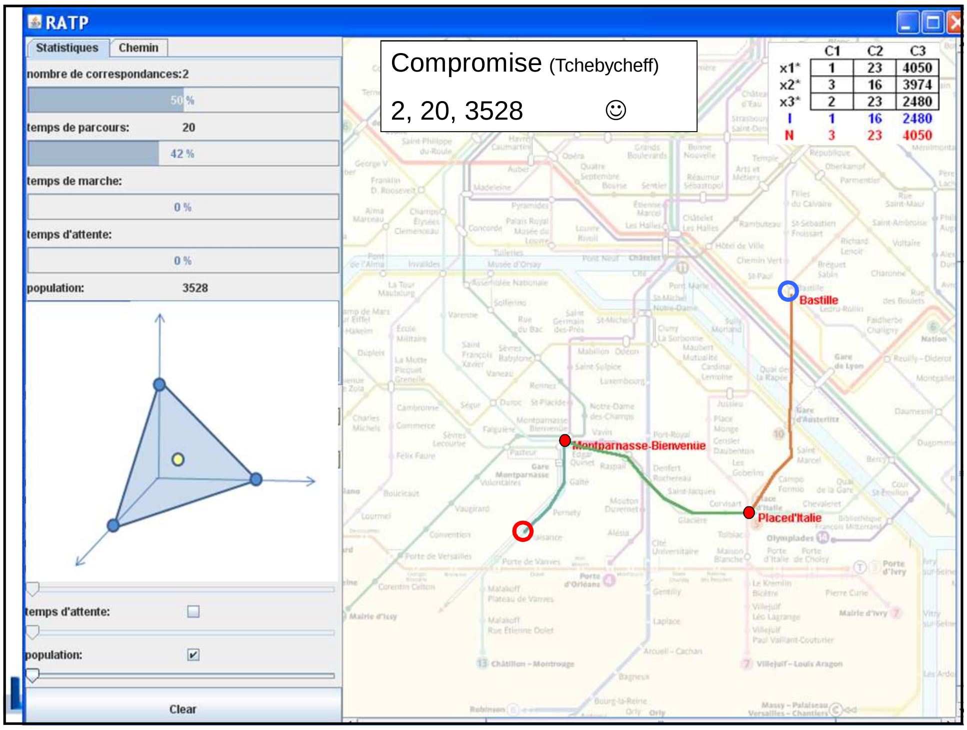
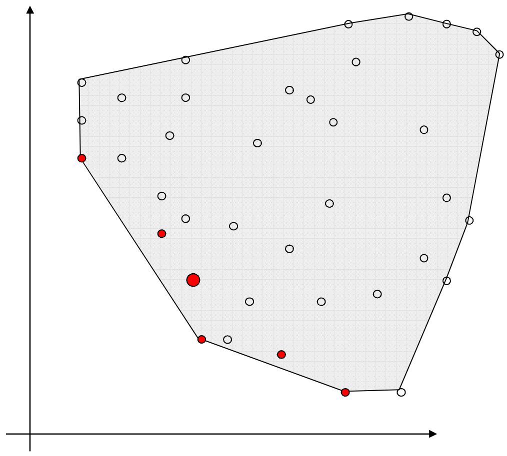
Statistiques
Chemin
nombre de correspondances:1
100 %
temps de parcours: 23
0 %
temps de marche:
0 %
temps d'attente:
0 %
population: 4050
0 %
meilleur compromis
Eppstein
RDW
Chercher
chemin suivant
Départ:
Plaisance
Arrivée:
Bastille
nombre de correspondances:
temps de parcours:
temps de marche:
temps d'attente:
population:
Clear
Champs-Elysées-Clemenceau
C1 C2 C3
x1* 1 23 4050
x2*
x3*


RATP
Statistiques
Chemin
nombre de correspondances:3
0 %
temps de parcours: 16
100 %
temps de marche:
0 %
temps d'attente:
0 %
population: 3974
4 %
meilleur compromis
Eppstein
RDW
Chercher
chemin suivant
Départ:
Plaisance
Arrivée:
Bastille
nombre de correspondances:
temps de parcours:
temps de marche:
temps d'attente:
population:
Clear
Chatelet
HoteldeVille
Montparnasse-Bienvenue
C1 C2 C3
x1* 1 23 4050
x2* 3 16 3974
x3*


RATP
Statistiques
Chemin
nombre de correspondances:2
50 %
temps de parcours: 23
36 %
temps de marche:
0 %
temps d'attente:
0 %
population: 2480
100 %
meilleur compromis
Eppstein
RDW
Chercher
chemin suivant
Départ:
Plaisance
Arrivée:
Bastille
nombre de correspondances:
temps de parcours:
temps de marche:
temps d'attente:
population:
Clear
C1 C2 C3
x1* 1 23 4050
x2* 3 16 3974
x3* 2 23 2480
I 1 16 2480
N 3 23 4050
Bastille
Duroc
Gared'Austerlitz


RATP
Statistiques
Chemin
nombre de correspondances:2
50 %
temps de parcours: 20
42 %
temps de marche:
0 %
temps d'attente:
0 %
population: 3528
temps d'attente:
population:
Clear
Compromise (Tchebycheff)
2, 20, 3528 ☺
C1 C2 C3
x1* 1 23 4050
x2* 3 16 3974
x3* 2 23 2480
I 1 16 2480
N 3 23 4050
Bastille
Montparnasse-Bienvenue
Placed'Italie

Compromise search in multiobjective (combinatorial) optimization



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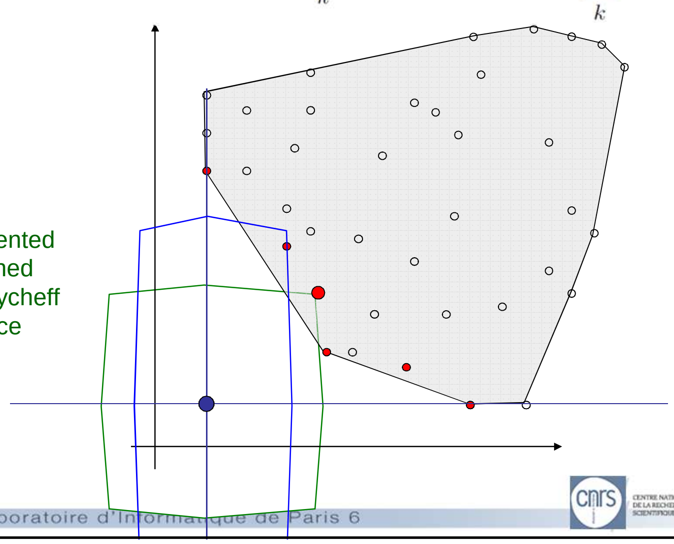
Compromise search in multiobjective (combinatorial) optimization

$$\text{minimize} f(x, \lambda, r) = \max_k \{\lambda_k |x_k - r_k|\} + \varepsilon \sum_k \lambda_k |x_k - r_k|$$

Augmented Weigthed Tchebycheff distance



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EX 2: Fairness in multiagent assignment problems

2 reviewers per paper

u_{ij}	*Paper 1*	*Paper 2*	*Paper 3*	*Paper 4*	*Paper 5*
Reviewer 1	3	3	4	3	4
Reviewer 2	3	4	4	2	3
Reviewer 3	1	2	3	2	3

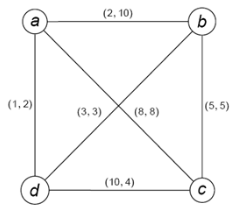
Solution 1 $1 \leftarrow \{1,3,4,5\}$ $2 \leftarrow \{1,2,3\}$ $3 \leftarrow \{2,4,5\}$

$x = (14, 11, 7)$ $\Sigma=32$ min = 7

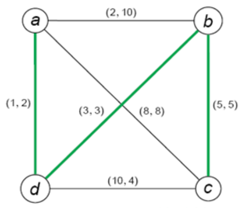
Solution 2 $1 \leftarrow \{1,4,5\}$ $2 \leftarrow \{1,2,3\}$ $3 \leftarrow \{2,3,4,5\}$

$x = (10, 11, 10)$ $\Sigma=31$ min = 10

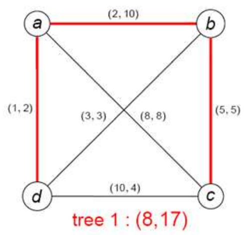
EX3: Robustness and optimization under total uncertainty



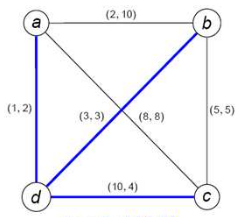
Kouvelis and Yu (80), Vincke (99)



robust tree : (9,10)



tree 1 : (8,17)



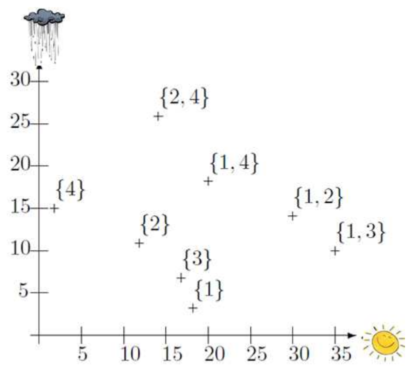
tree 2 : (14,9)

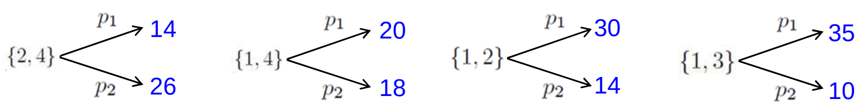
EX4: Knapsack problem under risk

EX4: Knapsack problem under risk

k	1	2	3	4
y_{1k}	18	12	17	2
u_{2k}	3	11	7	15
w_k	4	5	6	5

$$\begin{array}{ll} \max & z_1 = 18x_1 + 12x_2 + 17x_3 + 2x_4 \\ \max & z_2 = 3x_1 + 11x_2 + 7x_3 + 15x_4 \\ s.t. & 4x_1 + 5x_2 + 6x_3 + 5x_4 \leq 10 \\ & x_i \in \{0,1\}, i = 1,\ldots,4 \end{array}$$





EX4: Knapsack problem under risk

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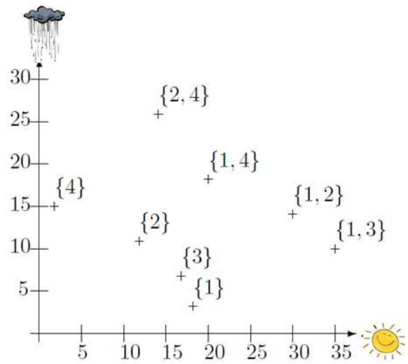
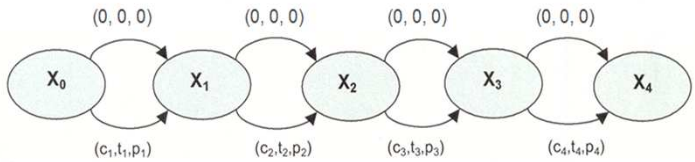


Also a vector-valued shortest path problem



Preference-based combinatorial optimization

- combinatorial structure of the set of alternatives
- multidimensional evaluation of solutions $(x_1, \dots, x_n)$
  - *multicriteria analysis* : x_i performance w.r.t criterion i
  - *multiagent decision-making* : x_i satisfaction of agent i
  - *decision under uncertainty* : x_i consequence in scenario s_i

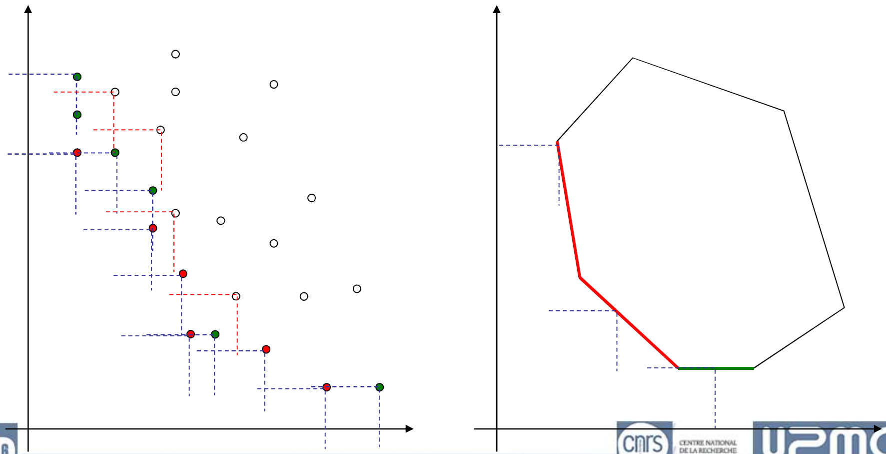
Tools

- preference models and optimality concepts
- algorithms to find preferred solutions on combinatorial domains

→ Algorithmic Decision Theory

Basic preference models: Pareto and weak Pareto Optimality

$$x < y \iff \begin{cases} \forall i = 1, \dots, n, & x_i \le y_i \\ \exists i \in \{1, \dots, n\}, & x_i < y_i \end{cases}$$



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Some references in multiobjective optimization

Main topic : determination of the set of Pareto Optimal Solutions

- *shortest paths :* Vincke (75), Hansen (80), Martins (84), Warburton (87), Stewart and White (91), Gandibleux et al. (06)
- *spanning trees :* Corley (83), Serafini (86), Hamacher and Ruhe (94), Andersen (96), Ramos and al. (98), Zhou and Gen (99), Knowles and Corne (01), Steiner and Radzik (06), Sourd and Spanjaard (08)
- *knapsack :* Ulungu and Teghem (97), Climaco, Figueira and Martins (01), Captivo et al. (03), Kumar and Barnajee (06), Bazgan, Hugot and Vanderpooten (08), Perny and Spanjaard (08)
- *assignment :* R Malhotra et al. (82), D. Tuyttens, J. Teghem and Ph. Fortemps (00)
- *scheduling :* Tkindt (02)
- Books, surveys : Ehrgott (00), Ehrgott and Gandibleux (00,04)

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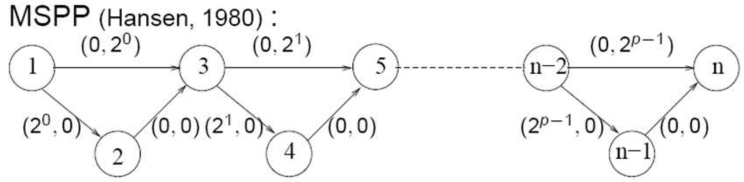
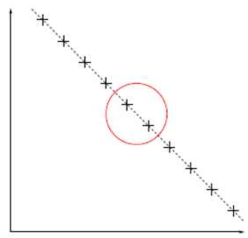
Pareto-optimal paths: an intractable problem

MSPP (Hansen, 1980) :



$$\forall P,\ c_1(P) + c_2(P) = \sum_{i=0}^{p-1} 2^i$$

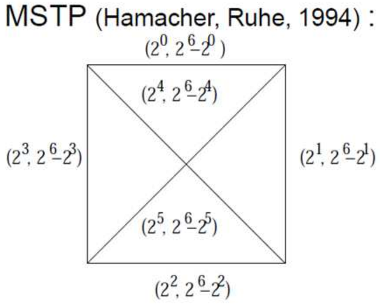


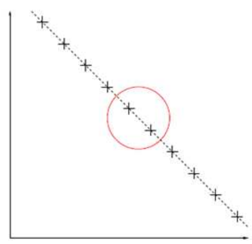
The number of Pareto-optimal solutions exponentially grows with the size of the graph (number of nodes)

Pareto-optimal trees: an intractable problem

Number of non-dominated solutions can be exponential in $|V|$

MSTP (Hamacher, Ruhe, 1994) :





$$\forall T,\ c_1(T) + c_2(T) = (n-1)2^m$$

Preference models for vector optimization

DOMINANCE RELATIONS

- Pareto dominance (MCDM/Social Choice)
- $\varepsilon-$dominance (MOCO)
- Lorenz dominance (Equity measurement)
- Second Order Stochastic Dominance (Risk-aversion)

SCALARIZING FUNCTIONS

- Weighted sum
- Additive utility (MAUT), EU (risk)
- Max, Leximax (Bottleneck, Robust optimization)
- Tchebycheff norm (MCDM)
- OWA, WOWA (Fairness, MCDM)
- Yaari's model, RDU (Risk)
- Choquet integral (Uncertainty, MCDM)

Preference-based optimization: a research program

	Pareto	ε-Pareto	Lorenz	SSD	EU	Tcheb	OWA	WOWA	RDU	Choquet
Paths										
Trees										
Assign										
Planning										
Knapsack										

Approximation of Pareto-optimal paths

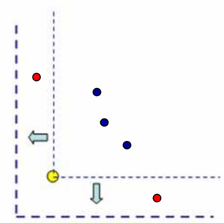
	Pareto	ε-Pareto	Lorenz	SSD	EU	Tcheb	OWA	WOWA	RDU	Choquet
Paths										
Trees										
Assign										
Planning										
Knapsack		1								

Approximated dominance

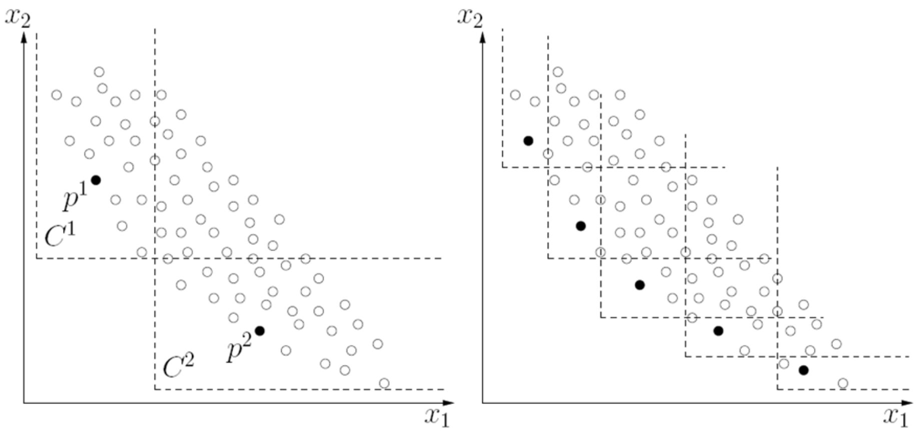
DEFINITION (ε-PARETO DOMINANCE)

Pour tout $\varepsilon > 0$ *et* $x, y \in X$,

$$x \precsim_\varepsilon y \Leftrightarrow [\forall i \in N, x_i \leq (1+\varepsilon) y_i]$$



Approximation = covering of the Pareto set



Main result

DEFINITION: $\preccurlyeq_\varepsilon$-COVERING

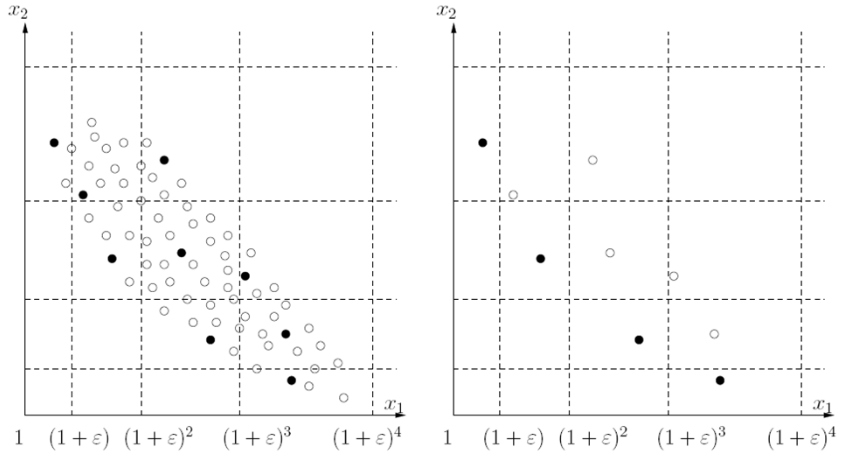
A subset $Y \subseteq X$ is said to be a $\preccurlyeq_\varepsilon$-covering of X if for all $x \in X$ there exists $y \in Y$ such that $y \preccurlyeq_\varepsilon x$.

THEOREM (PAPADIMITRIOU & YANNAKAKIS, 2000)

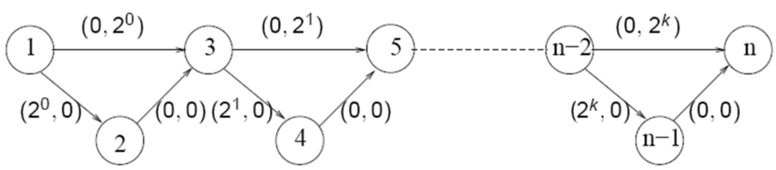
For any fixed number $m > 1$ of objectives, for any finite $\varepsilon > 0$ and any set X of bounded vectors x such that $1 \leq x_i \leq M$ for all $i \in \{1, \ldots, m\}$, there exists a $\preccurlyeq_\varepsilon$-covering subset of X the size of which is polynomial in $\log M$ and $1/\varepsilon$.

Existence of covering with bounded size (PY00)

$(1 + \lfloor \log M / \log(1+\varepsilon) \rfloor)^m$ $\quad$ $(1 + \lfloor \log M / \log(1+\varepsilon) \rfloor)^{m-1}$



An example using Hansen's graphs



$$|P_\varepsilon| \leq \left\lfloor \frac{\log 2^{k+1}}{\log(1+\varepsilon)} \right\rfloor + 1$$

Example : $k = 15$ et $\varepsilon = 0.1$

- $2^{16} = 65536$ Pareto optimal elements
- $\left\lfloor \frac{\log 65536}{\log(1.1)} \right\rfloor + 1 = 117$ (upper bound)

Application to biobjective knapsack problems

$$\max \sum_{j=1}^{n} p_{1j} x_j, \quad \max \sum_{j=1}^{n} p_{2j} x_j$$
$$\text{subject to } \sum_{j=1}^{n} w_j x_j \leq b$$
$$x_j \in \{0,1\} \; \forall j \in \{1,\ldots,n\}$$

Project selection, product design, team configuration, resource allocation…

n	30	40	50	60	70	80
	MOA*					
time	0.397	1.879	11.31	43.66	215.2	457.7
ε	FPTAS					
0.005	0.353	1.514	7.922	29.90	127.8	226.5
0.01	0.297	1.077	4.842	18.29	65.91	97.26
0.05	0.046	0.036	0.065	0.331	0.555	0.393
0.1	0.003	0.001	0.001	0.002	0.001	0.001
ε	MOA^*_ε					
0.005	0.315	0.940	4.225	18.58	62.37	110.4
0.01	0.179	0.364	1.389	9.294	19.75	35.11
0.05	0.008	0.007	0.013	0.064	0.065	0.075
0.1	0.001	0.001	0.001	0.001	0.001	0.001

Table 1. Numerical results on the biobjective knapsack.

[Perny et Spanjaard, ECAI'08]

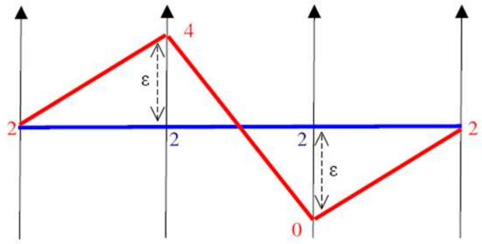
Lorenz-optimal paths

	Pareto	ε-Pareto	Lorenz	SSD	EU	Tcheb	OWA	WOWA	RDU	Choquet
Paths			2							
Trees										
Assign										
Planning										
Knapsack		1								

Aim: favouring well-balanced cost distributions

DEFINITION (TRANSFER PRINCIPLE)

Let $x \in \mathbb{R}^m_+$ s.t. $x_i > x_j$ for some $i, j \in \{1, \ldots, m\}$:
for all ε such that $0 < \varepsilon < x_i - x_j$: $x - \varepsilon e_i + \varepsilon e_j \succ x$



Generalized Lorenz dominance

DEFINITION (L-DOMINANCE)

$\forall x, y \in \mathbb{R}^m_+,\ x \succeq_L y \iff L(x) \succeq_P L(y)$

where $L(x) = (x_{(1)}, x_{(1)} + x_{(2)}, \ldots, x_{(1)} + x_{(2)} + \ldots + x_{(m)})$
with $x_{(1)} \geq x_{(2)} \geq \ldots \geq x_{(m)}$

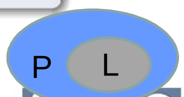
(10, 10, 10) $>_L$ (12, 9, 10) because (10, 20, 30) $>_P$ (12, 22, 31)

THEOREM (HARDY, LITTLEWOOD AND POLYA, 1929, CHONG, 1976)

For all $x, y \in \mathbb{R}^m_+$, if $x \succ_P y$, or if x obtains from y by an admissible transfer then $x \succ_L y$.

Conversely, if $x \succ_L y$, there exists a sequence of admissible transfers and/or Pareto improvements to transform y into x.

- Lorenz dominance refines Pareto dominance
- Favours well-balanced solutions (transfer principle)



Complexity issues for L-optimal paths

PROPOSITION

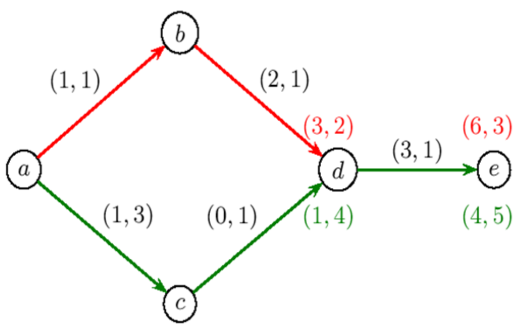
The problem of finding L-efficient paths in a graph is, in worst case, intractable, i.e. it requires a number of operations which grows exponentially with the size of the instance.

PROPOSITION

Deciding whether there exists a path whose cost distribution L-dominates a given cost-vector is an NP-complete decision problem.

The same results hold for spanning tree

L-optimal paths and the Bellman principle



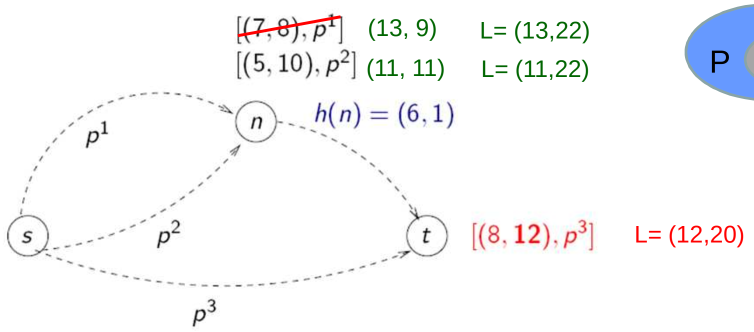
$(3,2) \succ_L (1,4)$ $\quad$ $(4,5) \succ_L (6,3)$

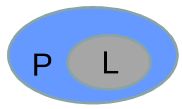
L=(3, 5) L=(4, 5) $\quad$ L=(5,9) L= (6,9)

A simple label-setting algorithm based on MOA*

Focused multiobjective search (paths)

- A Multiobjective version of Dijkstra Algorithm [Martins'84,StewWhite'91]
- prune subpaths L-dominated by an already detected solution path.





Numerical tests for L-optimal paths

(random instances, graph density ~ 50%)

m	$\#nodes$	# L-opt	time (s)
2	1000	2.20	0.12
	3500	2.25	1.75
	6000	2.45	5.75
5	1000	5.10	0.25
	3500	5.70	4.14
	6000	6.60	13.69
10	1000	10.75	0.55
	3500	14.15	9.47
	6000	13.5	30.97

[Perny and Spanjaard, UAI'03]

Refining Lorenz Dominance: OWA

THEOREM (PERNY, SPANJAARD, STORME 06)

$\succsim$ satisfies Neutrality, L-monotonicity, complete weak-order, continuity and independence iff it is reprensetable by an OWA function with strictly decreasing weights $w_i > w_{i+1}$ for all i.

$$OWA(x_1, \ldots, x_n) = \sum_{i=1}^{n} w_i x_{(i)}$$

with $x_{(1)} \geq x_{(2)} \geq \ldots \geq x_{(n)}$

x is strictly preferred to y   iff   OWA(x) < OWA(y)

Example: w1=3/6   w2=2/6   w3=1/6

OWA(10, 10, 10) = 10

OWA(4, 16, 10) = (48+20+4)/6 = 72/6=12

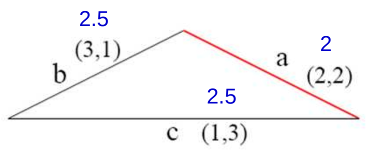
The OWA-optimal spanning tree problem

	Pareto	ε-Pareto	Lorenz	SSD	EU	Tcheb	OWA	WOWA	RDU	Choquet
Paths			2							
Trees							3			
Assign										
Planning										
Knapsack		1								

OWA-optimal spanning tree problem

NP-hard in general (includes min-max optim)

Failure of the greedy approach



f(x, y) = 0.75 max{x, y} + 0.25 min{x, y}

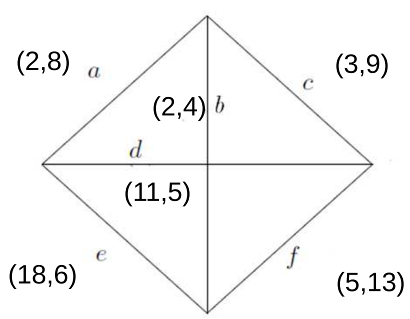
OWA optimal edge: a (2, 2)

Completion: a ∪ b (5, 3)   a ∪ c (3, 5)   OWA =4.5

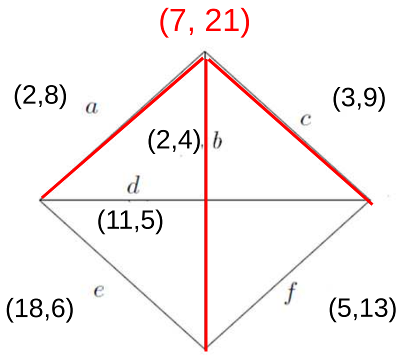
b ∪ c is clearly better with (4, 4)   OWA =4

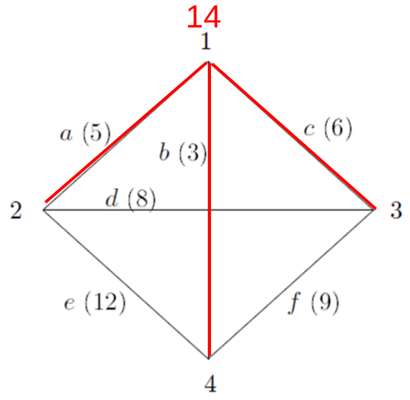
An instance of the OWA-optimal spanning tree problem

Min W(x, y) = 0.75 max{x, y} + 0.25 min{x, y}



Key property if $\forall i,\ w_i > w_{i+1}$ then $\sum_{i=1}^{n} w_i x_{(i)} \geq \frac{1}{n} \sum_{i=1}^{n} x_i$





0.75 max{7, 21} + 0.25 min{7, 21} = 17.5 ≥ 14

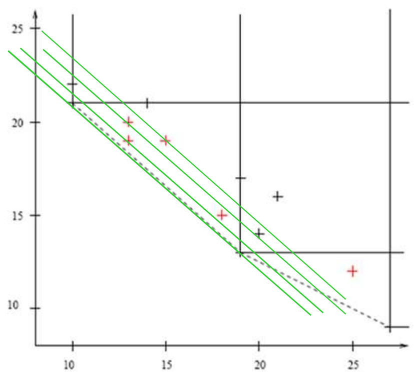
The ranking approach for OWA optimal ST

Linear scalarizing function :

$$f_\omega(x) = \sum_{i=1}^{q} \omega_i x_i$$

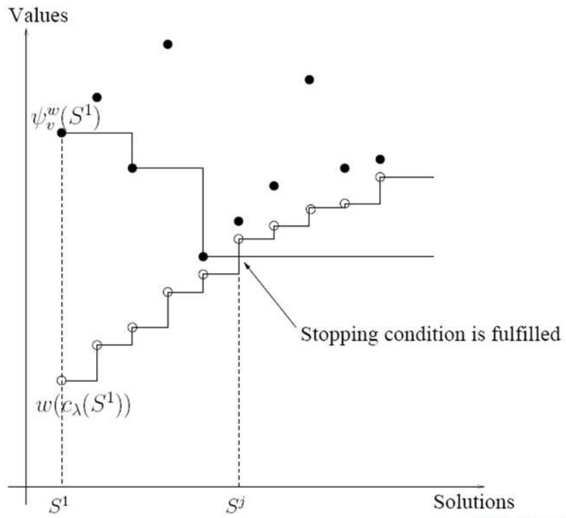
- optimization
+ ranking

- k-minimum spanning trees (Gabow, 77) or Ibaraki, Katoh, Mine (1981) : $O(km + min(n^2, m \log\log n))$



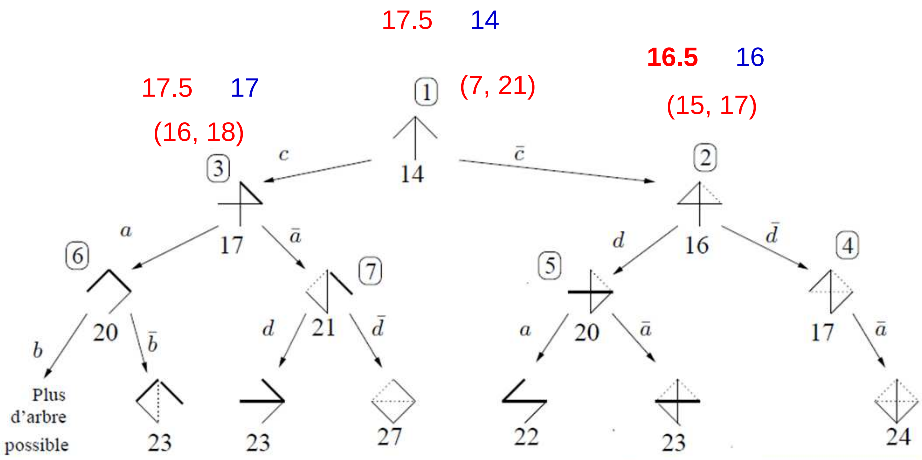
Requires a stopping conditions

Stopping condition for ranking algorithms



OWA-optimal spanning tree

An example with : W(x, y) = 0.75 max{x, y} + 0.25 min{x, y} ≥ 0.5 x + 0.5 y

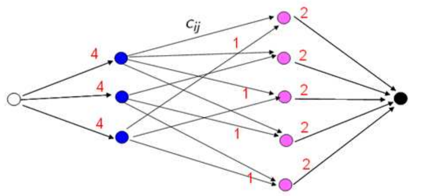


Choquet-optimal assignment for multi-agents fair allocation problems

(conf. paper assignement, task allocation, Santa Claus pb)

	Pareto	ε-Pareto	Lorenz	SSD	EU	Tcheb	OWA	WOWA	RDU	Choquet
Paths			2							
Trees							3			
Assign							4			
Planning										
Knapsack		1								

Fair assignment problems



$$\text{Min} \quad W(x) = \sum_{k=1}^{n} w_i x_{(i)} \qquad w_1 > w_2 > \ldots, w_n > 0$$

$$s.t. \begin{cases} x_i = \sum_{j=1}^{m} c_{ij} z_{ij} & i = 1, \ldots, n \\ l'_j \leq \sum_{i=1}^{n} z_{ij} \leq u'_j & j = 1, \ldots, m \\ l_i \leq \sum_{j=1}^{m} z_{ij} \leq u_i & i = 1, \ldots, n \\ z_{ij} \in \{0, 1\} & \forall i, \forall j \end{cases}$$

As soon as m > 1, finding an OWA optimal assignment is NP-hard

A MIP formulation for fair cost minimization

$y = (y_1, \dots, y_n)$ $\quad y_{(1)} \geq y_{(2)} \dots \geq y_{(n)}$

$$\mathrm{OWA}(y) = \sum_{k=1}^{n} w_i y_{(k)} = \sum_{k=1}^{n} w'_k L_k(y) \qquad w' = (w_1 - w_2, \dots, w_{n-1} - w_n, w_n) > 0$$

$L_k(y) =$ $\quad \mathrm{Max} \sum_{i=1}^{n} \alpha_i^k y_i$, $\sum_{i=1}^{n} \alpha_i^k = k$, $0 \leq \alpha_i^k \leq 1 \quad i = 1 \dots n$

dual: $\mathrm{Min}\ kr_k + \sum_{i=1}^{n} b_i^k$, $r_k + b_i^k \geq y_i$, $b_i^k \geq 0 \quad i = 1 \dots n$

$$\mathrm{Min} \sum_{k=1}^{p} w'_k \left(kr_k + \sum_{i=1}^{n} b_i^k \right)$$

$r_k + b_i^k \geq y_i$
$b_i^k \geq 0$

Ogryczak, 03

Final MIP Formulation

$$\mathrm{Min} \sum_{k=1}^{n} w'_k \left(k \times r_k + \sum_{i=1}^{n} b_{ik} \right)$$

$$\text{s.t.} \begin{cases} l'_j \leq \sum_{i=1}^{n} z_{ij} \leq u'_j & j = 1, \dots, m \\ l_i \leq \sum_{j=1}^{m} z_{ij} \leq u_i & i = 1, \dots, n \\ r_k + b_{ik} \geq \sum_{j=1}^{m} c_{ij} z_{ij} & \forall i, k = 1, \dots, n \\ b_{ik} \geq 0 & \forall i, k \\ z_{ij} \in \{0, 1\} & \forall i, \forall j \end{cases}$$

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Numerical tests with Cplex for OWA assignment

n =	100	200	300	400	500	600	700	800	900
t (OWA)	.98	2.37	10.6	23.0	32.4	57.7	84.5	158	227

Times (in seconds) for fair assignment problems with n agents, costs in {1, …, 20}

n =	100	200	300	400	500	600	700	800	900	1000	1100
t	.23	1.58	4.8	10	20	37	57	93	151	222	361

Times (in seconds) for paper assignment problems with n reviewers, 3n papers costs in {1, …, 5}, matrix density 20%, max nb of paper per agent = 5.

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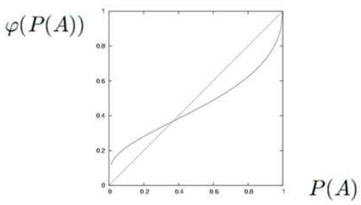
RDU and Planning under Risk

	Pareto	ε-Pareto	Lorenz	SSD	EU	Tcheb	OWA	WOWA	RDU	Choquet
Paths			2							
Trees							3			
Assign							4			
Planning									5	
Knapsack		1								

RDU, a generalization of EU

(Rank Dependent Expected Utility, Quiggin 81)

$$(x_1, \ldots, x_n)$$



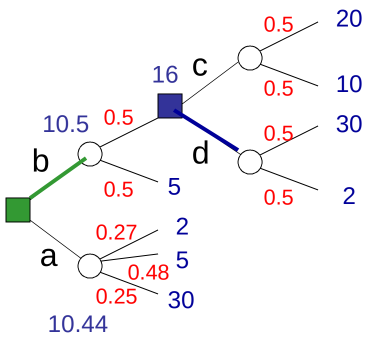
$$u(x_i) \leq u(x_{i+1})$$

$$RDU(x) = \sum_{i=1}^{n} \left[\varphi(\sum_{k=i}^{n} p(x_k)) - \varphi(\sum_{k=i+1}^{n} p(x_k)) \right] u(x_i)$$

Non linear with respect to probabilities and payoffs

Fits to preferences observed in Allais and Ellsberg Examples

Dynamic Decision Problems are also combinatorial problems



EU(a) = 2*0.27+ 5*0.48 + 30* 0.25 = 10.44

EU(bc) = 5*0.5+10*0.25 + 20*0.25 = 10

EU(bd) = 5*0.5+ 2*0.25 + 30*0.25 = 10.5

Backward induction

EU(c) = 10*0.5 + 20*0.5 = 15

EU(d) = 30*0.5 + 2*0.5 = 16

Computational issues for RDU-optimal policies

$$RDU(x) = \sum_{i=1}^{n}\left[\varphi(\sum_{k=i}^{n} p(x_k)) - \varphi(\sum_{k=i+1}^{n} p(x_k))\right] u(x_i)$$

$$\varphi(\ p(A)\ ) = \sqrt{-\ln(p(A))}$$

An example of dynamic inconsistency

RDU(a) = 2 + (5-2) φ(0.75) + (30-5) φ(0.25) = 11.41

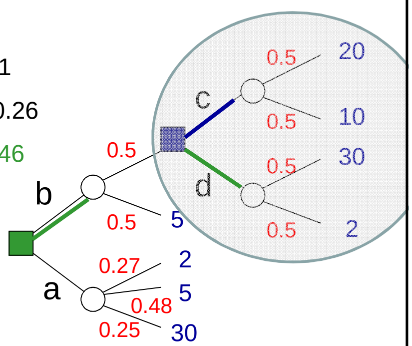
RDU(bc) = 5 + (10-5) φ(0.5) + (20-10) φ(0.25) = 10.26

RDU(bd) = 2 + (5-2) φ(0.75) + (30-5) φ(0.25) = 11.46

But !

RDU(c) = 10 + (20-10) θ(0.5) = 14.35

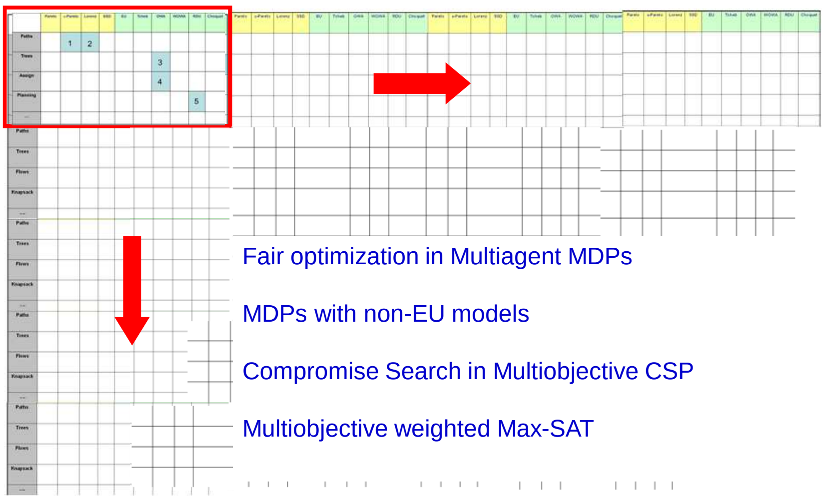
RDU(d) = 2 + (30-2) θ(0.5) = 14.18



Conclusion (main messages)

- Decision theory has provided various sophisticated models to go beyond Pareto dominance in different contexts
- Preference-based optimization with such models is significantly harder than in the classical case
- Classical approaches used for simple linear criteria and for Pareto Optimization do not extend easily (e.g. dynamic programming, greedy approaches)
- New problems requiring new algorithmic solutions
- Cross-fertilization of decision theory and (combinatorial) optimization, a challenging program for the future!

Still some work to do…



Fair optimization in Multiagent MDPs

MDPs with non-EU models

Compromise Search in Multiobjective CSP

Multiobjective weighted Max-SAT

Some related papers of the decision team at LIP6

LP Solvable Models for Multiagent Fair Allocation problems
Julien Lesca, n; Perny, Patrice; ECAI'10 pp. 387-392

Infinite order Lorenz dominance for fair multiagent optimization
Golden, Boris; Perny, Patrice, AAMAS'10, 383-390

Choquet-based optimisation in multiobjective shortest path and spanning tree problems
Galand Lucie, Perny, Patrice; Spanjaard, Olivier, EJOR (2010), vol. 204, pp. 303–315.

Near Admissible Algorithms for Multiobjective Search
Perny, Patrice; Spanjaard, Olivier; ECAI'08, pp. 490-494

Search for Choquet-optimal paths under uncertainty
Galand, Lucie; Perny, Patrice, UAI'07, pp. 125-132

State Space Search for Risk-averse Agents
Perny, Patrice; Spanjaard, Olivier; Storme, Louis-Xavier; IJCAI'07, pp. 2353-2358

A decision-theoretic approach to robust optimization in multivalued graphs
Perny, Patrice; Spanjaard, Olivier; Storme, Louis-Xavier;
Annals of Operations Research (2006) Vol. 147, 1, pp. 317-341

Search for Compromise Solutions in Multiobjective State Space Graphs
Galand, Lucie; Perny, Patrice; ECAI'06, pp. 93-97.

A preference-based approach to spanning trees and shortest paths problems
Perny, Patrice; Spanjaard, Olivier; EJOR (2005).